

TWIN PARADOX

Contemplate a scenario with twins Alex and Steven. Steven is in on the planet Earth and he observes Alex on a rocket ship moving past with speed v (v must be near c for the effect to be significant).

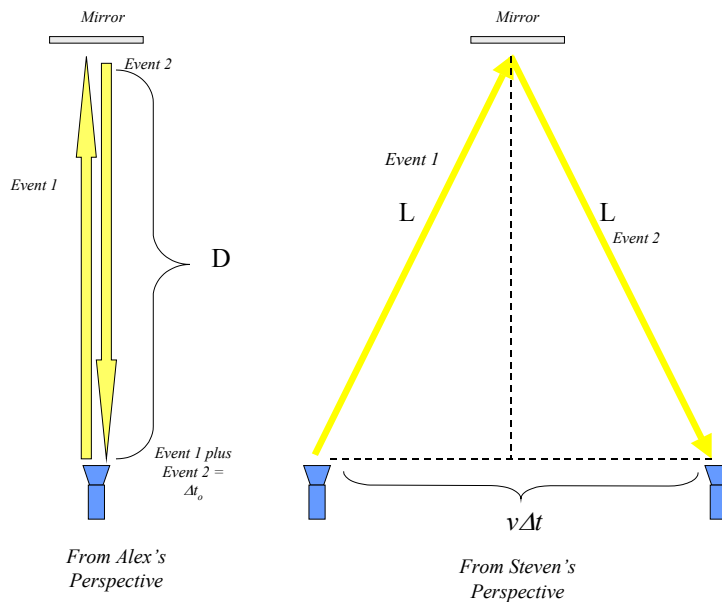
Alex has a clock, a flashlight, a mirror and a very accurate and specialized clock on the rocket ship with him as shown below. A pulse of light leaves the flashlight and is reflected by the mirror back towards the flashlight.

Alex can use the specialized clock to measure the time between the light leaving the flashlight and returning back to him. This time interval of Alex's clock is given by

$$\Delta t_0 = 2D/c$$

Note: As such, $D = c(\Delta t_0/2)$

All of Alex's equipment is at rest with respect to Alex's rocket ship. Note: Δt_0 is the time between two events: Event #1 is the flash of light leaving the flashlight and contacting the mirror and event #2 is the flash of light returning from the mirror.



How do the same two events appear to Steven who is on Earth? The time interval Δt Steven observes between event #1 and event #2 is

$$\Delta t = [2L/c]$$

Note: As such, $L = [c\Delta t/2]$

From Steven's perspective, the mirror moves a horizontal distance $v(\Delta t)$ between event #1 and event #2. The time interval Δt observed by Steven is longer than the time interval Δt_0 Alex observes because $L > D$. How much larger can easily be calculated using the Pythagorean theorem.

$$[v\Delta t/2]^2 + D^2 = L^2$$

Previously

$$D = c(\Delta t_0/2) \quad \& \quad L = [c\Delta t/2]$$

As such:

$$[v\Delta t/2]^2 + c^2(\Delta t_0/2)^2 = [c\Delta t/2]^2$$

Which can be simplified to:

$$[v\Delta t]^2 + c^2(\Delta t_0)^2 = [c\Delta t]^2$$

$$v^2(\Delta t)^2 + c^2(\Delta t_0)^2 = c^2(\Delta t)^2$$

$$c^2(\Delta t_0)^2 = c^2(\Delta t)^2 - v^2(\Delta t)^2 = (\Delta t)^2 [c^2 - v^2]$$

$$(\Delta t)^2 = c^2(\Delta t_0)^2 / [c^2 - v^2] = (\Delta t_0)^2 / [1.0 - v^2/c^2]$$

$$\Delta t = \Delta t_0 / [1.0 - (v^2/c^2)]^{1/2}$$

Which is the Time Dilation Relationship we're looking for.